

$$d = \frac{a_m - a_n}{m - n} \Rightarrow d = \frac{a_{10} - a_7}{10 - 7} = \frac{21 - 7}{3} = \frac{14}{3} = 2$$

۱

$$a_7 = 7 \Rightarrow a_1 + 6d = 7 \xrightarrow{d=2} a_1 + 12 = 7 \Rightarrow a_1 = -5$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \Rightarrow S_{10} = \frac{10}{2}(2 \times (-5) + 9 \times 2) = 5 \times 22 = 110$$

$$a_n = 2(2)^{1-n} \Rightarrow \begin{cases} a_1 = 2 \times 2^0 = 2 \\ a_7 = 2 \times 2^{-6} = \frac{1}{16} \end{cases}$$

۲

$$q = \frac{a_7}{a_1} = \frac{\frac{1}{16}}{2} = \frac{1}{32}$$

$$S_n = \frac{a_1(q^n - 1)}{q - 1} \Rightarrow S_6 = \frac{2\left(\left(\frac{1}{32}\right)^6 - 1\right)}{\frac{1}{32} - 1} = \frac{2\left(\frac{1}{2^{48}} - 1\right)}{-\frac{31}{32}} = \frac{2\left(-\frac{2^{48}-1}{2^{48}}\right)}{-\frac{31}{32}} = \frac{2^{48}-1}{81}$$

$$\begin{cases} 3, 8, 13, 18, \dots \Rightarrow d = 5 \\ 1, 4, 7, 10, 13, \dots \Rightarrow d = 3 \end{cases}$$

۳

اولین جمله مشترک: $a_1 = 13$

$$\Rightarrow a_n = a_1 + (n-1)d \Rightarrow a_n = 13 + (n-1)(15) \Rightarrow a_n = 15n - 2$$

$d = 15$: ک م م قدرنسبت ها

$$100 \leq 15n - 2 < 300 \xrightarrow{+2} 102 \leq 15n < 302 \xrightarrow{\div 15} 6/8 \leq n < 20/1$$

پس جملات از جمله هفتم تا جمله بیستم را با هم جمع می‌کنیم که تعداد جملات ۱۴ است.

$$S_{14} = \frac{14}{2}(a_7 + a_{20}) = 7(103 + 298) = 2807$$

$$b - d, b, b + d$$

برای حل این سؤال جمله وسط را b در نظر می‌گیریم.

۴

$$b - d + b + b + d = 18 \Rightarrow 3b = 18 \Rightarrow b = 6$$

مجموع این سه جمله:

ضرب این سه جمله:

$$(b-d)b(b+d) = -168 \xrightarrow{b=6} (6-d)6(6+d) = -168 \xrightarrow{\div 6} 36 - d^2 = -28$$

$$\Rightarrow d^2 = 64 \Rightarrow d = \pm 8$$

چون دنباله افزایشی است $d = 8$ در نظر می‌گیریم.

$$\xrightarrow{b=6} -2, 6, 14, \dots \Rightarrow \begin{cases} a_1 = -2 \\ d = 8 \end{cases} \Rightarrow a_n = a_1 + (n-1)d \Rightarrow a_n = -2 + (n-1)(8)$$

$$\Rightarrow a_n = 8n - 10 \xrightarrow{n=12} a_{12} = 96 - 10 = 86$$

بنابراین جمله دوازدهم برابر ۸۶ است.

$$a_1 + a_r + a_0 + \dots + a_{1q} = 13\omega \Rightarrow 13\omega = S_{10} = \frac{10}{r}(ra_1 + 9(rd)) \Rightarrow ra_1 + 18d = 13$$

$$a_r + a_{r-1} + a_{r-2} + \dots + a_1 = 15 \Rightarrow 15 = S_{10} = \frac{10}{r}(ra_r + 9(rd)) \Rightarrow 5(r(a_1 + d) + 18d) = 15$$

$$\Rightarrow ra_1 + 2 \cdot d = 3$$

$$\begin{cases} ra_1 + 18d = 13 \\ ra_1 + 2 \cdot d = 3 \end{cases} \Rightarrow a_1 = \frac{1}{r}, d = \frac{2}{r}$$

$$\begin{cases} S_{10} = 13 \Rightarrow 13 = \frac{10 \cdot (ra_1 + 9rd)}{r} \\ S_{100} = 13 \cdot 10 + 13 = 143 \Rightarrow 143 = \frac{100 \cdot (ra_1 + 99rd)}{r} \end{cases}$$

$$\Rightarrow \begin{cases} 13 = ra_1 + 99d \\ 143 = ra_1 + 999d \end{cases} \Rightarrow 130 = 900d \Rightarrow d = \frac{1}{3}$$

$$13 = ra_1 + 99 \Rightarrow -90 = ra_1 \Rightarrow a_1 = -\frac{90}{r}$$

$$\left. \begin{cases} \alpha + \beta = -\frac{b}{a} = \sqrt{r} \\ \alpha\beta = \frac{c}{a} = -r \end{cases} \right\} \Rightarrow \frac{\alpha + \beta}{\alpha^r + \beta^r} = \frac{\alpha + \beta}{(\alpha + \beta)^r - r\alpha\beta} = \frac{\sqrt{r}}{(\sqrt{r})^r - r(-r)} = \frac{\sqrt{r}}{r}$$

$$\alpha + \beta = -\frac{b}{a} = r \text{ و } \alpha\beta = \frac{c}{a} = 1$$

$$\alpha^r + \beta^r + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta)^r - r\alpha\beta + \frac{\alpha + \beta}{\alpha\beta}$$

$$A = (r)^r - r(1) + \frac{r}{1} = 18$$

$$S = \alpha + \beta = \frac{r}{r}$$

$$P = \alpha \cdot \beta = -\frac{1}{r}$$

$$\text{معادله } \alpha \Rightarrow rx^r - rx - 1 = 0 \xrightarrow{x=\alpha} r\alpha^r - r\alpha - 1 = 0 \Rightarrow r\alpha^r - r\alpha = 1$$

$$r\alpha^r + r\alpha - \beta^r + r\beta = r\alpha^r - \alpha^r + r\alpha - r\alpha + r\beta - \beta^r$$

$$= (r\alpha^r - r\alpha) - (\alpha^r + \beta^r) + r(\alpha + \beta) = 1 - \frac{r}{r} + \frac{r}{r} = \frac{1 - r + r}{r} = \frac{1}{r}$$

$$x^r - rx - r = 0 \xrightarrow{x=\alpha} \alpha^r - r\alpha - r = 0 \Rightarrow \alpha^r = r\alpha + r$$

$$\alpha + \beta = \frac{-b}{a} = \frac{r}{1} = r$$

$$r\alpha^r + r\beta + r = r(r\alpha + r) + r\beta + r = r\alpha + r\beta + 1r + r = r(\alpha + \beta) + 1r$$

$$= r \times r + 1r = 18$$